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Computational Analysis of Weakly Magnetized Plasma's Effects on Electromagnetic Wave Propagating Perpendicular to the Magnetic Field

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Abstract

Magnetised plasma influences electromagnetic wave propagation, this resulting in polarisations at different velocities, an effect that can result in ghost signals. This work provided critical information on the effects of magnetized plasma on electromagnetic (EM) wave propagation perpendicular to the magnetic field. Electromagnetic waves propagating perpendicular to the magnetic field in a weakly magnetized plasma were studied. Furthermore, there are two modes of EM wave propagation perpendicular to the field: ordinary (O) and extraordinary (X). The values of the ordinary (O) mode's frequency were obtained by substituting the values of its wave number (k) into its dispersion relation, while the same frequency values were substituted to obtain the values of its refractive index (η_O^2). The plasma frequency values derived from the Bohm-Gross formulae for electron plasma. The plasma frequency values obtained from the Bohm-Gross formulae for electron plasma waves were substituted into the extraordinary mode dispersion relation to obtain frequency values with cut-off frequencies at ω_L and ω_R. The refractive index values were calculated using the same frequency values (X-mode). The results revealed that for EM wave propagation perpendicular to magnetic field in a weakly magnetized plasma in vacuum, the speed as well as the direction was unperturbed. The results also revealed that the properties of electromagnetic waves propagating perpendicular to magnetic field in a weakly magnetized plasma in an ordinary (O) mode were different from those for extraordinary (X) mode, with the former having a higher phase speed than the latter. Additionally, we discovered that, the phase velocity (υP) exceeded the speed of light (c), which is consistent with the basic ideas of special relativity.

Keywords: Plasma, Electromagnetic, Wave, Propagation and Frequency

I. Introduction

In contrast to the other three fundamental states of matter (solid, liquid, and gas), plasmas are collections of free charged particles that exhibit quasi-neutrality and collective interactions (Shukla, 2012; Lichtenberg et al, 2005; Atul et al., 2022). The fundamental difference between solids, liquids, and gases is the strength of the bonds that hold their constituent particles together. These binding forces are relatively strong in solids, weak in liquids, and almost non-existent in gases (Bittencourt, 2004). The kinetic energy per plasma particle must exceed the ionizing potential of atoms in order for matter to transition to its fourth state and exist as a plasma (Umran & Marek, 2011).

A dispersion can brand one of the many possible modes of wave propagation in plasma (Bittencourt, 2004). Plasma can be created artificially by applying a strong magnetic field to a neutral gas or by heating it (Atul et al., 2022; Umran & Marek, 2011). Plasmas are created artificially by applying heat or strong electric fields (Atul et al., 2022; Umran & Marek, 2011). Plasma can be created in the laboratory (the generated plasma can be steady or transient, stable or unstable, low or high temperature, low or high density), in a gas discharge (an electric field applied across the ionized gas accelerates the mobile electrons to energies robustly high enough to ionize other atoms by collisions), and by thermodynamics (the process involves raising the temperature of a substance until a satisfactorily high fractional ionization is obtained) (Bittencourt, 2004). Ionization can also be caused by x-rays or gamma rays, which have much shorter wavelengths (Bittencourt, 2004). Plasma matter becomes electrically conductive due to gas ionization and extremely high kinetic energy. Plasma properties are similar to those of gases in that they have no definite shape or volume. The overall charge of plasma is nearly zero (Atul et al., 2022). An anisotropic medium is one in which the electrical and/or magnetic properties depend on the direction of field vectors (Yakhno &Çiçek, 2014). Gilbert-damping can be used to reduce the amplitude and velocity of loop-solitons during propagation, resulting in slower decay rates than those found in linear theory for multi-solitons (Jin & Lin, 2020). Through cyclotron resonance, the axial magnetic field enhances nonlinear coupling, while the density ripple provides phase synchronism (Kumar et al., 2010). Waves are disturbances in the plasma that can transport energy through space and time (Steinvall, 2022). The vibration of an electric charge produces electromagnetic waves (NASA, 2010). Electric fields are critical in governing plasma formation and controlling when, where, and how energy is deposited in the plasma environment (Barnat, 2019).

(2019, Mikitchuk) A method for determining the distribution and evolution of electron density and temperature is required for the calculation of various plasma properties (thermal pressure, electrical conductivity, plasma frequency, and so on) and for understanding the physical processes that occur in tweaks. Plasmas are most commonly used in the control of thermonuclear fusion reactions, lighting and plasma display panels, technology by producing integrated circuits, rocket engine propulsion systems, converting the kinetic energy of a dense plasma flowing across a magnetic field into electrical energy, waste processing, selectively killing bacteria and viruses, and weld materials (Umran & Marek, 2011; Lichhtenberg et al, 2005).

An external magnetic field can be applied perpendicular to the discharge current to generate a strong electric field that acts to accelerate ions. Such devices are widely used in electric propulsion (Kaganovich et al., 2021).

The application of a magnetic multipole cusp field around the port of a helicon plasma source significantly increases the downstream plasma density (Varberg & Fredriksen, 2019).

Depending on the orientation of the magnetic field with respect to the propagation direction, the EM wave has several passes and stop bands in a magnetized plasma (Mandal et al., 2021). In this

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study, the external magnetic field is chosen to be perpendicular to both the wave propagation direction and the electric field of the EM wave, which is the X mode.

II. Methodology

Ordinary (O-Mode) and extraordinary (X-Mode) modes of EM wave propagation perpendicular to magnetic field were investigated in a weakly magnetized plasma. The values of the O-frequency mode were obtained by plugging the wave number (k) into the dispersion relation, and the same frequency values were used to calculate the refractive index squared (η_O^2). The plasma frequency values obtained from the Bohm-Gross formulae for electron plasma waves were substituted into the dispersion relation to obtain the same frequency values with cutoff at ω_L and ω_R.. The same frequency values were entered into their respective refractive index squared dispersion equations to obtain their respective refractive index squared $(\eta_L^2 \text{ and } \eta_R^2)$.

Theoretical Equations

The analysis of electron frequency is usually based on the mathematical equation of the form

$$\omega_e = e\vec{B}/2\pi M_e \quad (1)$$

The statistical dispersion relation for ordinary wave is given by (Alho & Kallio, 2016;Bellan, 2004):

$$\omega^2 = \omega_{pe}^2 + k^2c^2 \quad (2)$$

The refractive index for ordinary mode with cut-off frequency at $\omega = \omega_{pe}$ and resonance at $\omega = 0$ is given by the relation:

$$\eta^2 = \frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega^2} \quad (3)$$

$$\eta^2 = 1 - \frac{\omega_e^2}{\omega^2} \quad (3a)$$

$$\eta^2 = \frac{k^2c^2}{\omega^2} \quad (3b)$$

The Refractive index for extraordinary mode can be obtained by the mathematical relation (Alho & Kallio, 2016):

$$\eta^2 = \frac{k_x^2c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega^2}\left(\frac{\omega^2 - \omega_{pe}^2}{\omega^2 - \omega_{UH}^2}\right) \quad (4)$$

It follows from equation (4)

$$\eta^2 = 1 - \frac{\omega_e^2}{\omega^2}\left(\frac{\omega^2 - \omega_{pe}^2}{\omega^2 - \omega_{UH}^2}\right) \quad (4a)$$

Also,

$$k_x^2 = \frac{\eta^2\omega^2}{c^2} \quad (4b)$$

where:

ω_{UH} (upper hybrid frequency) *is expressed as:*

$$\omega_{UH}{}^2 = \omega_e^2 + \Omega_e^2 \quad (5)$$

Ω_e (electron cyclotron frequency) is given by:

$$\Omega_e = \frac{eB}{M_e} \qquad (6)$$

The evaluation of extraordinary wave travelling to the left or right direction is usually based on the mathematical equation of the form (Maikasuwa et al, 2019):

$$\omega_{\binom{L}{R}} = \pm\frac{\Omega_e}{2} + \left(\omega_{pe}^2 + \frac{\Omega_e^2}{4}\right)^{1/2} \qquad (7)$$

It follows

The evaluation of extraordinary wave travelling to the left direction

$$\omega_L = -\frac{\Omega_e}{2} + \left(\omega_{pe}^2 + \frac{\Omega_e^2}{4}\right)^{1/2} \qquad (7a)$$

The evaluation of extraordinary wave travelling to the right direction

$$\omega_R = +\frac{\Omega_e}{2} + \left(\omega_{pe}^2 + \frac{\Omega_e^2}{4}\right)^{1/2} \qquad (7b)$$

The Bohm-Gross Formula for electron plasma waves is related as (Maikasuwa et al,. 2019*;* Andreev et al,.2000*)*:

$$\omega^2 = \omega_{pe}^2 + k^2\gamma V_t^2 \qquad (8)$$

where;

$\gamma = (N+2)/N$: The adiabatic index (9)

Since plane waves are one-dimensional perturbations (i.e., the plasma is compressed in the $\hat{k}$ direction only), $N = 1. \therefore \gamma = 3$ (Bellan, 2004).

k: is the wave number and is given by the relation:

$$k = \frac{2\pi}{l}\left(\eta_x^2 + \eta_y^2 + \eta_z^2\right) \qquad (10)$$

The phase velocity is given by the theoretical relation (Chen, 1994; Goldston & Rutherford 1995;Umran & Marek, 2011; Steinvall, 2022):

$$\upsilon_p = \frac{\omega}{k} = c\left(1 + \omega_p^2/c^2k^2\right)^{1/2} > c \qquad (11)$$

It follows

$$\upsilon_p = \frac{\omega}{k} > c \qquad (11a)$$

$$\upsilon_p = c\left(1 + \omega_p^2/c^2k^2\right)^{1/2} > c \qquad (11b)$$

The group velocity is given by the mathematical relation (Shvets et al., 2012; *Steinvall, 2022*):

Theoretical Equation Computation

The Wave Number $(\mathbf{k})$

Imploring equation (10) the wave number for ordinary mode is mathematically calculated as:

$$k = \frac{2\pi}{l}\left(\eta_x^2 + \eta_y^2 + \eta_z^2\right) \qquad where \quad l = 1$$

If $\eta_x = 0$, $\eta_y = 0$ and $\eta_z = 0$;

$$k = \frac{2\pi}{l}\left(\eta_x^2 + \eta_y^2 + \eta_z^2\right) = \frac{2\pi}{l}\left((0)^2 + (0)^2 + (0)^2\right) = 0.000$$

$\therefore k = 0.000$

For the various values $\eta_x = 0, \eta_y$, and η_z. The results are as follows:

$\therefore k = 0.00, 6.28, 12.57, 18.85$

Bohm-Gross Formula for Electron Plasma Waves

The Bohm-Gross formular for electron frequency is given by equation (8) as:

$$\omega^2 = \omega_{pe}^2 + k^2\gamma V_t^2$$

where:

Electron frequency, $\omega_{pe} = \frac{eB}{2\pi M_e} = \frac{1.60\times10^{-19}\times0.2}{2\pi\times9.11\times10^{-31}} = 5.59 \times 10^9 \text{ rad/s}$

(E
lectron charge, $e = 1.60 \times 10^{-19}$ C; Magnetic field, $B = 0.2$ T; Mass of elctron, $M_e = 9.11 \times 10^{-31}$ kg
and Speed of light, $c = 3 \times 10^8 \text{ ms}^{-1}$*)*.

Imploring equation (8)

$\omega^2 = \omega_{pe}^2 + k^2\gamma V_t^2$.

where $\omega_{pe} = 5.59 \times 10^9$ rad/s, $\gamma = 3$ *and* $V_t = c = 3 \times 10^8 \text{ ms}^{-1}$

when $k = 0.00$

$\omega^2 = \omega_{pe}^2 + k^2\gamma V_t^2 = (5.59 \times 10^9)^2 + 3(0)^2(3 \times 10^8)^2$

$\therefore \omega = 6.47 \times 10^9$ rad/s

The Bohm-Gross frequency values are, respectively, $\omega = 0.56 \times 10^{10}$ rad/s, 0.65×10^{10} rad/s, 0.86×10^{10} rad/s and 1.13×10^{10} rad/s *for* various values of $k = 0.00, 6.28, 12.57, 18.85$.

- EM Wave Propagation Perpendicular to Magnetic Field (Ordinary Mode)

Equation (2) states that the dispersion relation for the ordinary mode ($\vec{E} \perp \vec{B}$), describes the ordinary wave in the EM field:

$\omega_O^2 = \omega_{pe}^2 + k^2c^2$

when $k = 0.00$; $\omega_O^2 = (5.59\ 1 \times 10^9)^2 + (0)^2(3 \times 10^8)^2$

$\therefore \omega_O = 5.591 \times 10^9$ rad/s

According to the same tend

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The frequency values for an EM wave propagating perpendicular to a magnetic field in the ordinary mode are therefore: $\omega_O = 5.59 \times 10^9\ \mathrm{rad/s}, 5.90 \times 10^9\ \mathrm{rad/s}, 6.74 \times 10^9\ \mathrm{rad/s}, 7.95 \times 10^9\ \mathrm{rad/s}$.

The refractive index dispersion relation (ordinary mode) is given by equation (3a) as follows:

$$\eta_O^2 = \mathbf{1} - \frac{\boldsymbol{\omega}_{\mathbf{pe}}^2}{\boldsymbol{\omega}_O^2}$$

Using equation (3b), equation $(3b)$; $\eta_O^2 = 1 - \frac{\omega_e^2}{\omega_O^2} = \frac{\omega_O^2 - \omega_e^2}{\omega_O^2}$

$$\eta_O^2 = \frac{(\mathbf{5.590 \times 10^9})^2 - (\mathbf{5.591 \times 10^9})^2}{(\mathbf{5.590 \times 10^9})^2} = 0.00$$

$\therefore \eta_O^2 = 0.00$

Continuing the trend at various values of ω_O.

$\eta_O^2 = 0.00, 0.10, 0.31, 0.51$

- ❖ EM Wave Propagation Perpendicular to Magnetic Field (X-Mode)

i. Extraordinary(x) mode with Cut-off at $\boldsymbol{\omega_L}$

The dispersion frequency for an extraordinary (x) mode of wave propagating to the left is given by equation (7) and is equal to:

$$\omega_L = -\frac{\Omega_e}{2} + \left(\omega_{pe}^2 + \frac{\Omega_e^2}{4}\right)^{1/2} \qquad (7a)$$

Bohm-Gross frequency values are as follows: $\omega = 0.5591 \times 10^{10}\ \mathrm{rad/s}, 0.6474 \times 10^{10}\ \mathrm{rad/s}, 0.860 \times 10^{10}\ \mathrm{rad/s}$ and $1.128 \times 10^{10}\ \mathrm{rad/s}$

respectively.

Equation $(7a)$ can be used to determine the dispersion of refractive index squared for extraordinary mode propagation to the left

$\omega_L = -\frac{\Omega_e}{2} + \left(\omega_{pe}^2 + \frac{\Omega_e^2}{4}\right)^{1/2}$ where $\Omega_e = \frac{eB}{M_e} = \frac{1.60 \times 10^{-19} \times 0.2}{9.11 \times 10^{-31}} = 3.51 \times 10^{10}\ \mathrm{rad/s}$ and $\omega_{pe} = 5.59 \times 10^9\ \mathrm{rad/s}$.

when $\omega = 0.559 \times 10^{10}\ \mathrm{rad/s}$;

$$\omega_L = -\frac{3.513 \times 10^{10}}{2} + \left[(0.559 \times 10^{10})^2 + \frac{(3.51 \times 10^{10})^2}{4}\right]^{1/2} = 0.87 \times 10^9\ \mathrm{rad/s}$$

$\therefore \omega_L = 0.8681 \times 10^9\ \mathrm{rad/s}$

Following the same way

Therefore, the cut-off frequency for X-mode with cut-off frequency at ω_L are:

$\omega_L = 0.87 \times 10^9\,\mathrm{rad/s}, 1.15 \times 10^9\,\mathrm{rad/s}, 1.99 \times 10^9\,\mathrm{rad/s}, \times 10^9\,\mathrm{rad/s}, 3.31 \times 10^9\,\mathrm{rad/s}$.

From equation (5)

$$\omega_{UH} = \sqrt{\omega_{pe}^2 + \Omega_e^2} = \sqrt{(5.59 \times 10^9)^2 + (3.52 \times 10^{10})^2} = 3.56 \times 10^{10}\ \mathrm{rad/s}$$

Equation (4a) can be used to determine the refractive index squared dispersion for extraordinary mode propagation to the left as follows:

$$\eta_L^2 = 1 - \frac{\omega_{pe}^2}{\omega_L^2}\left(\frac{\omega_L^2 - \omega_{pe}^2}{\omega_L^2 - \omega_{UH}^2}\right)$$

when $\omega_L = 0.868 \times 10^9\,\mathrm{rad/s}$;

$$\eta_L^2 = 1 - \frac{(5.591 \times 10^9)^2}{(0.868 \times 10^{10})^2}\left(\frac{(0.868 \times 10^{10})^2 - (5.591 \times 10^9)^2}{(0.868 \times 10^{10})^2 - (3.564 \times 10^{10})^2}\right) = 0.00$$

We have the following results for different values of $\omega_L = 1.15 \times 10^9\,\mathrm{rad/s}, 1.99 \times 10^9\,\mathrm{rad/s}, \times 10^9\,\mathrm{rad/s}, 3.31 \times 10^9\,\mathrm{rad/s}$ we have:

With the cut-off frequency at ω_L, the squares of the refractive index values for extraordinary mode propagation to the left are as follows:

$$\eta_L^2 = 0.00, 0.45, 0.83, 0.95$$

The wave number for X-mode propagation to the left can be calculated *equation (4b)* as follows:

$k_L^2 = \frac{\eta_L^2 \omega_L^2}{c^2}$ *where* $c = 3 \times 10^8 ms^{-1}$

when $\omega_L = 0.87 \times 10^9\,\mathrm{rad/s}$ *and* $\eta_L^2 = 0.00$;

$$k_L^2 = \frac{(0.00) \times (0.87 \times 10^9)^2}{(3 \times 10^8)^2}$$

$\therefore k_L = 0.00$

Therefore, $k_L = 0.00, 2.57, 6.04, 10.75$ was the wave number for the X-mode with the cut-off at ω_L.

ii. Extraordinary (x) Mode with Cut-off at $\boldsymbol{\omega_R}$

Equation (7b) gives the dispersion relation for the propagation of EM waves to the right as Using equation $(7b)$, the dispersion relation EM wave propagation to the right

$$\omega_R = +\frac{\Omega_e}{2} + \left(\omega_e^2 + \frac{\Omega_e^2}{4}\right)^{1/2}$$

The Bohm-Gross frequency values: $\omega = 5.59 \times 10^9\, \mathrm{rad/s}, 6.74 \times 10^9\, \mathrm{rad/s}, 8.60 \times 10^9\, \mathrm{rad/s}$ and $11.44 \times 10^9\, \mathrm{rad/s}$ respectively.

With respect to the Bohm-Gross frequency values:
when $\omega = 0.559 \times 10^{10}\, \mathrm{rad/s}$;

$$\omega_R = \frac{3.513 \times 10^{10}}{2} + \left[(0.559 \times 10^{10})^2 + \frac{(3.51 \times 10^{10})^2}{4}\right]^{1/2} = 3.600 \times 10^{10}\, \mathrm{rad/s}$$

$\therefore\ \omega_R = 3.599 \times 10^{10}\, \mathrm{rad/s}$.

The cut-off frequency (ω_R) for extra-ordinary mode is $\omega_R = 3.600 \times 10^{10}\, \mathrm{rad/s}, 3.635 \times 10^{10}\, \mathrm{rad/s}, 3.709 \times 10^{10}\, \mathrm{rad/s}, 3.850 \times 10^{10}\, \mathrm{rad/s}$, all of which follow the same trend.

The cut-off frequency (ω_R) for extra-ordinary mode, on the other hand, is $\omega = 35.997 \times 10^9\, \mathrm{rad/s}$ (i.e., when $k = 0.00$ and $\eta^2 = 0.00$). The frequency range that results from changing the Bohm-Gross frequency values in the equation for the extraordinary (x) mode with cut-off at ω_R is smaller than this.

Furthermore, using equation (4a), the refractive index squared (η^2) for extraordinary mode propagation to the right can be calculated:

$\eta_R^2 = 1 - \frac{\omega_e^2}{\omega_R^2}\left(\frac{\omega^2 - \omega_e^2}{\omega_R^2 - \omega_{UH}^2}\right)$ *where* $\omega_{UH} = 3.557 \times 10^{10}\, \mathrm{rad/s}$

when $\omega_R = 3.5997 \times 10^{10}\, \mathrm{rad/s}$

$$\eta_R^2 = 1 - \frac{(5.59 \times 10^9)^2}{(3.5997 \times 10^{10})^2}\left(\frac{(3.5997 \times 10^{10})^2 - (5.59 \times 10^9)^2}{(3.5997 \times 10^{10})^2 - (3.557 \times 10^{10})^2}\right) = 0.00$$

$\therefore \eta^2 = 0.00$

The same *pattern* was observed for various values of ω_R are:

$\eta_R^2 = 0.00, 0.01, 0.46, 0.72, 0.86$

The wave number for extraordinary mode propagation to the right can be calculated using Equation (4b), where $c = 3 \times 10^8\, \mathrm{ms}^{-1}$.

The wave number for propagation to the right is given by:

$$k_R^2 = \frac{\eta_R^2 \omega_R^2}{c^2}$$

when $\omega_R = 3.599 \times 10^{10}\, \mathrm{rad/s}$ and $\eta_R^2 = 0.965$

$$k_R^2 = \frac{(0.00)(3.599 \times 10^{10})^2}{(3 \times 10^8)^2}$$

$\therefore k_R = 0.00$

Following the same logic, the wave number values for extraordinary mode propagation to the right with cut-off frequency (ω_R) are:
$k_R = 0.00, 12.00, 81.78, 103.35, 115.083$

- The Phase Velocity

Equation (11)allows for the following computation of the phase velocity:

a. For O-Mode Propagation
According to equation $(11a)$*:*
$\upsilon_p = \frac{\omega}{k} > c$, the wave moves to the right and vice versa depending on the value of k.
when $\omega = 5.591 \times\ 10^9\, \mathrm{rad/s}$ *and* $k = 0.00$

$$\upsilon_{p0} = \frac{\omega}{k} = \frac{0.559 \times 10^{10}}{0} = \infty$$

Similarly, using different values of k as a result, using equation $(11a)$, the phase velocity for O-mode propagation is calculated as follows:
$\upsilon_{p0} = \infty, 9.3949 \times 10^8 ms^{-1}, 5.3644 \times 10^8 ms^{-1}, 4.2186 \times 10^8 ms^{-1}$. This implies that the phase velocity (υ_p) is greater than the speed of light (c).

a. X-Mode Propagation

i) For X-Mode Propagation to the Left

From equation $(11a)$
$\upsilon_p = \frac{\omega}{k} > c$, if ω/k is positive, the wave moves to the right and vice-versa.
when $\omega = \omega_L = 0.868 \times 10^9$ rad/s *and* $k_L = 0.00$

$$\upsilon_{pL} = \frac{\omega_L}{k_L} = \frac{0.87 \times 10^9}{0.00} = \infty$$

Following the same way
Therefore, the phase velocities for X-mode propagation to the left (using equation 11a) are obtained as:
$\upsilon_{pL} = \infty, 4.4942 \times 10^8 ms^{-1}, 3.2980 \times 10^8 ms^{-1}, 3.0791 \times 10^8 ms^{-1}$
This implies the phase velocity (υ_p) *was far greater than the speed of light* (c).

ii) For X-Mode Propagation to the Right

From equation $(11a)$
$\upsilon_p = \frac{\omega}{k} > c$, if ω/k is positive the wave moves to the right and vice-versa.
when $\omega = \omega_R = 35.997\ \times 10^9$ rad/s *and* $k = 0.00$

$$\upsilon_{pR} = \frac{\omega_R}{k_R} = \frac{35.99 \times 10^9}{0.00} = \infty$$

Following the same trend
Therefore, the phase velocities for X-mode propagation to the right (using equation 11a) is obtained as:
$\upsilon_{pR} = \infty, 3.000 \times 10^9 \mathrm{ms}^{-1}, 4.450 \times 10^8 \mathrm{ms}^{-1}, 3.7252 \times 10^8 \mathrm{ms}^{-1}, 3.3455 \times 10^8 \mathrm{ms}^{-1}$. This implies the phase velocity (υ_p) is far greater than the speed of light (c).

III. Results

Below are table of values of Refractive index, Wave number and Angular frequency.

Table 1: Dispersion values of Refractive Index and Wave Number – Equation (10)

η_X	η_y	η_z	K
0	0	0	0.00
0	0	1	6.28
0	1	0	6.28
1	0	0	6.28
1	1	0	12.57
1	0	1	12.57
0	1	1	12.57
1	1	1	18.85

Table 2: Values for Wave Numbers (k) and Refractive Indexes Square (η^2) with their associated frequencies.

Wave Number(k)				Frequency (ω), $\times 10^9$ rad/s				Refractive Index(η^2)		
Bohm -Gross	Ordinary (O) Mode	Extraordinary (X) Mode		Bohm-Gross	Ordinary (O) mode Cut-off frequency	Extraordinary (X) Mode Cut-off frequency		Ordinary (O) Mode	Extraordinary (X) mode	
(k)	(k_0)	(k_L)	(k_R)	(ω)	(ω_0)	(ω_L)	(ω_R)	(η_O^2)	(η_L^2)	(η_R^2)
0.00	0.00	0.00	0.00	5.591	5.591	0.868	35.997	0.00	0.00	0.00
6.28	6.28	2.57	12.00	6.474	5.900	1.155	36.000	0.10	0.45	0.01
12.57	12.57	6.04	81.78	8.600	6.743	1.992	36.350	0.31	0.83	0.46
18.85	18.85	10.75	103.35	11.280	7.952	3.310	37.090	0.51	0.95	0.72
–	–	–	115.08	–	–	–	38.500	–	–	0.86

Table 3: Comparison of phase velocities (υ_p) with speed of light (c)

Wave Number (k)			Speed of Light (c) (ms^{-1})	Phase Velocity $(\upsilon_p)(\times 10^8 ms^{-1})$		
Ordinary(O) Mode	Extraordinary(X) Mode			O-Mode	X-Mode	
K	k_L	k_R		υ_{pO}	υ_{pL}	υ_{pR}
0.000	0.00	0.00	3.00E8	∞	∞	∞
6.28	2.57	12.00	3.00E8	9.3949E8	4.49425E8	30.000E8
12.57	6.04	81.78	3.00E8	5.3644E8	3.2980E8	4.450E8
18.85	10.75	103.35	3.00E8	4.2186E8	3.0791E8	3.7252E8
–	–	115.08	3.00E8	–	–	3.3455E8

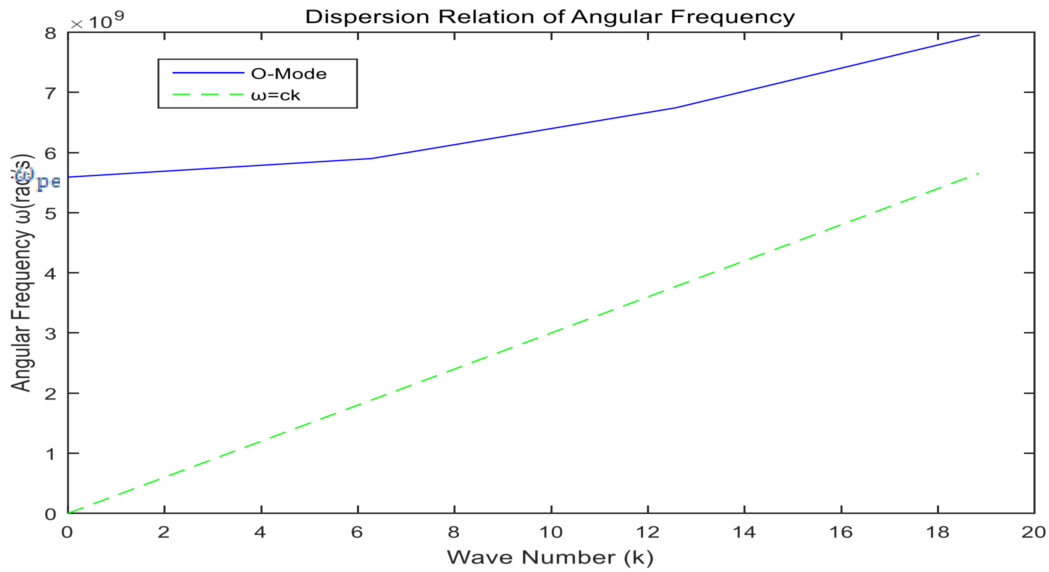

Figure I: Shows a graph of the phase velocity dispersion for ordinary mode propagation in a weakly magnetized plasma as a blue line, while the vacuum dispersion is shown as a green dotted

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line.

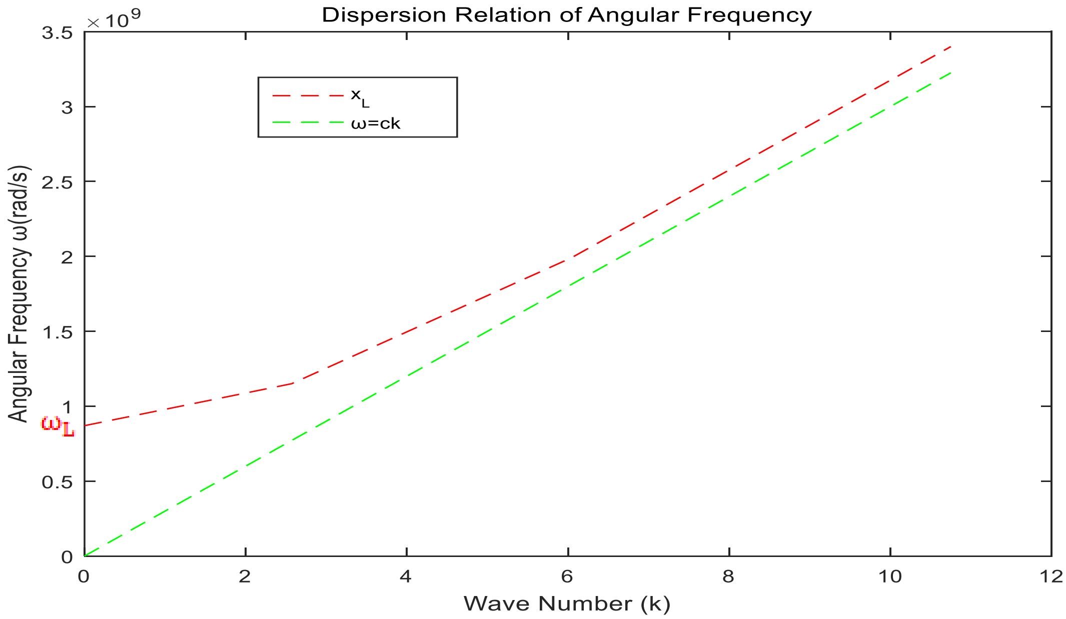

Figure II: Shows a graph of the phase velocity dispersion for extraordinary mode propagation to the left in a weakly magnetized plasma with a red dotted line and a green dotted line representing the dispersion in vacuum.

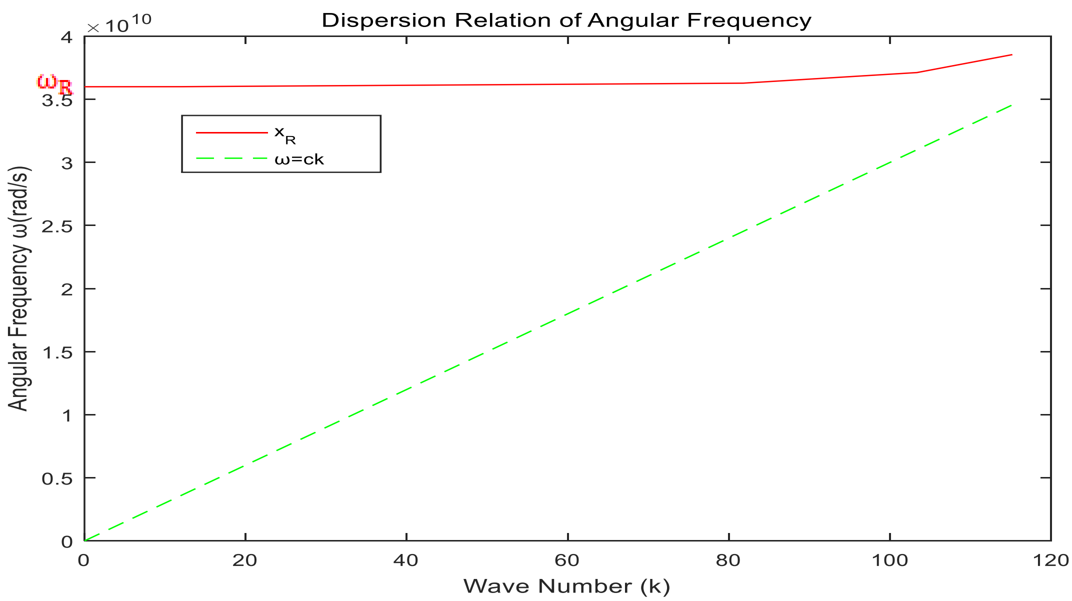

Figure III: Shows a graph of the phase velocity dispersion for extraordinary mode propagation to the right in a weakly magnetized plasma, with a red dotted line, and a green dotted line, representing the dispersion in vacuum.

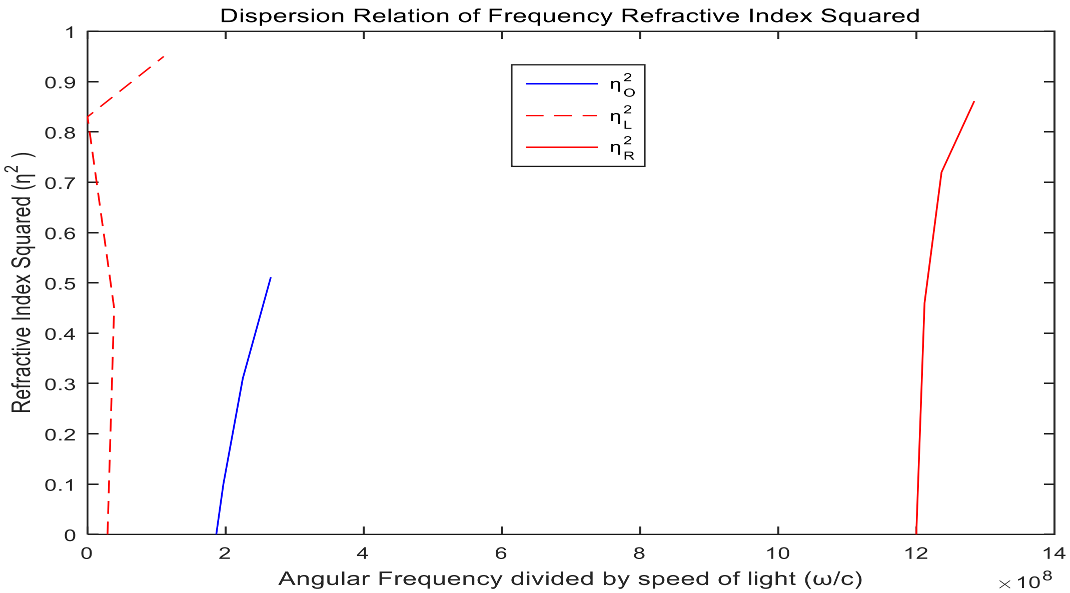

Figure IV: Shows the relationship between the Refractive Index squared(η^2)against the weighted average speed for ordinary (O-mode) and extraordinary (E-mode) propagation in a dispersive medium (red line and red dotted line).

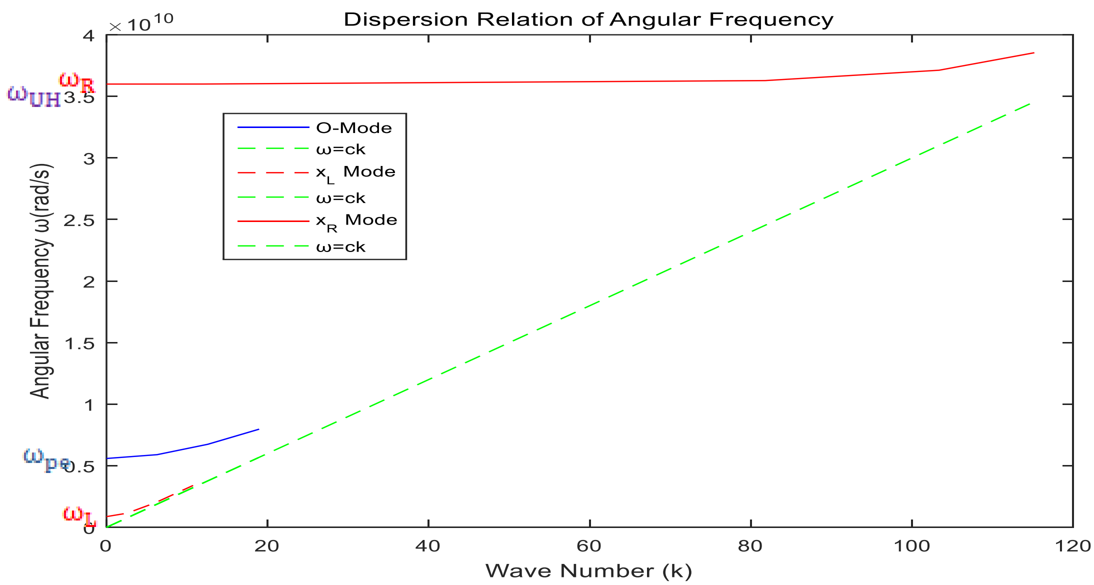

Figure V. Shows a sketch of the relationship between the angular velocity (ω) and wave number (k) of the electromagnetic wave ($\vec{E}\perp\vec{B}$)for the ordinary mode (O-Mode, shown in blue dotted line) and extraordinary mode (X-Mode, shown in red dotted line and red line, respectively, for propagation to the left and right, respectively) in a dispersive medium and in vacuum (green dotted line).

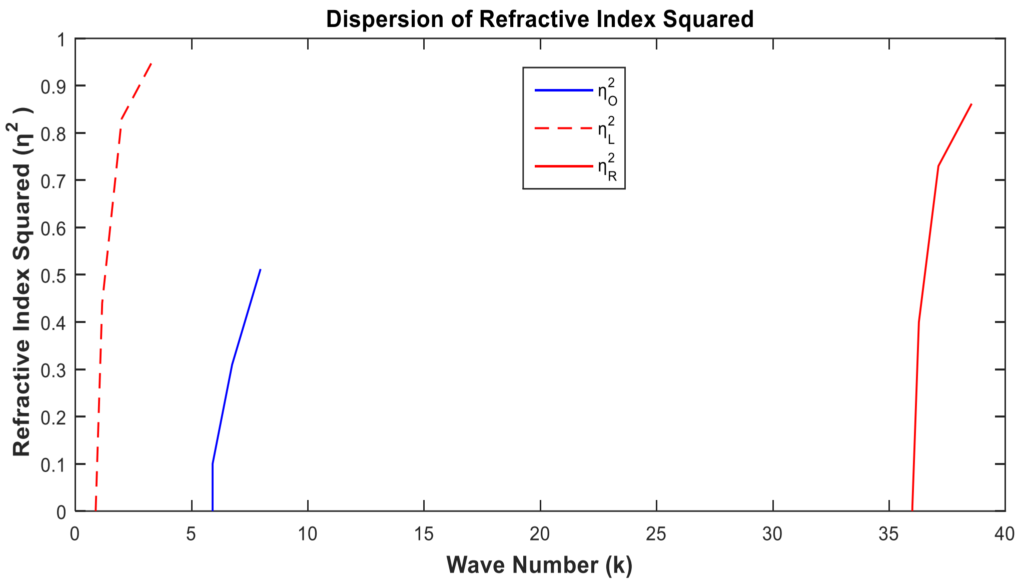

Figure VI: The dispersion of Refractive Index squared(η^2) against wave number (k) of EM Wave $(\vec{E}\perp\vec{B})$ for Ordinary Mode (O-Mode) propagation and Extraordinary Mode (X-Mode) propagation.

III. Discussion

As shown in column 2 of table 2, the frequency values for X-mode propagation with cut-off frequency at ω_L were less than those for ordinary (O) mode propagation with cut-off frequency at ω_{pe}, while the frequency for X-mode propagation with cut-off frequency at ω_R was greater than both of them.

Ordinary mode has the lowest refractive index at the peak stage, with a cut-off at ω_{pe}, followed by X-mode, which has a cut-off at ω_R. Meanwhile, as shown in column 3 of table 2, the refractive index of X-mode with cut-off at ω_L has attained the highest refractive index.

Figure (5) shows that as the frequency increased for X-wave, the phase velocity increased from c until the cut-off at ω_L was reached. The wave became evanescent as the frequency increased until it reached resonance at the upper hybrid frequency, ω_{UH}. It was then propagated until the second cut-off at ω_R. The bands where waves did not propagate were $(0 < \omega < \omega_O)$ for the O-mode, $(0 < \omega < \omega_L)$ and $(\omega_{UH} < \omega < \omega_R)$for the X-mode, and the other bands were pass bands.

The phase velocity (υ_p)exceeded the speed of light (c), as shown in sub column 1 of column 3 of table 3 above, which is typical for electromagnetic waves in plasma. Conversely, as the wave number (k) increased, the velocity is decreased, and vice versa.

The frequencies of EM wave propagation in vacuum were unaffected because they were linearly related (i.e., they are free to propagate themselves), as shown by the dotted green line in Figures (1,2, & 3), whereas in dispersive medium, such as plasma, different frequencies were observed propagating with dissimilar group velocities, as depicted in the same figures.

IV. Conclusion

In this research, we performed a detailed analysis of an EM wave propagating perpendicular to a magnetic field within a vacuum and a dispersive medium. As the medium being considered an anisotropic medium. In a magnetized plasma, the direction and magnitude of wave number (k) played an important role in the wave dynamics. As the wave number (k) increased, the phase and group velocities decreased and vice versa.

Indeed, we discovered that the phase velocity in plasma, exceeded the speed of light which is consistent with fundamental special relativity considerations.

Recommendations

In our analysis, electromagnetic waves were considered while ignoring ion motion and electrostatic ion waves, thereby ignoring their electromagnetic effects. As a result, we recommend that more research should be done to combine ion motion with electromagnetic effects where perpendicular Magneto sonic waves can be studied.

CONFLICT OF INTERESTS: None

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